Psym: Efficient Symbolic Exploration of Distributed Systems

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Verification of distributed systems using systematic exploration is daunting because of the many possible interleavings of messages and failures. When faced with this scalability challenge, existing approaches have traditionally mitigated state space explosion by avoiding exploration of redundant states (e.g., via state hashing) and redundant interleavings of transitions (e.g., via partial-order reductions). In this paper, we present an efficient symbolic exploration method that not only avoids redundancies in states and interleavings, but additionally avoids redundant computations that are performed during updates to states on transitions. Our symbolic explorer leverages a novel, fine-grained, canonical representation of distributed system configurations (states) to identify opportunities for avoiding such redundancies on-the-fly. The explorer also includes an interface that is compatible with abstractions for state-space reduction and with partial-order and other reductions for avoiding redundant interleavings. We implement our approach in the tool Psym and empirically demonstrate that it outperforms a state-of-the-art exploration tool, can successfully verify many common distributed protocols, and can scale to multiple real-world industrial case studies across Amazon.

CCS Concepts: • Computing methodologies → Distributed programming languages; • Software and its engineering → Formal software verification.

Additional Key Words and Phrases: distributed systems, systematic exploration, binary decision diagrams

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1 INTRODUCTION

Verification of distributed systems is challenging because of the need to reason about all possible behaviors resulting from a myriad interleavings of messages and failures. Existing approaches fall into two broad categories: interactive theorem-proving with specialized logics (e.g., [Jung et al. 2015; Sergey et al. 2018]), and automatic systematic exploration (e.g., [Desai et al. 2013a; Holzmann 1997; Lamport 2002]). Researchers have used theorem provers to construct correctness proofs for complex distributed systems [Hawblitzel et al. 2015; Padon et al. 2016; Wilcox et al. 2015], but this requires significant manual effort. In this paper, we are interested in mostly-automated systematic-exploration-based approaches that need less expert guidance. However, in practice, such approaches can achieve correctness guarantees only for small bounded instances of systems because of the state-space explosion problem, which results in poor scalability with increasing system complexity. Hence, there is a need for more efficient techniques that can push the limits on the complexity of systems that can be verified using systematic exploration.

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Explicit-state model-checkers (stateful explorers) (e.g., TLC [Yu et al. 1999], Zing [Andrews et al. 2004], and SPIN [Holzmann 1997]) have been widely successful in industry and academia for verification of distributed systems [Newcombe 2014; TLA+ 2023]. Most stateful explorers address the state explosion problem by using state caching (hashing) to avoid re-exploring already visited states. Explorers also use partial-order reduction (POR) [Clarke et al. 2001; Peled 2018] and its dynamic variants [Abdulla et al. 2014; Flanagan and Godefroid 2005; Nguyen et al. 2018; Tasharofi et al. 2012] to avoid exploring interleavings that are redundant due to independent transitions.

In distributed systems, transitions operate only on the local state of a process (no shared memory), and it is common to have multiple instances of the same process that execute the same code but have different local state (e.g., replicas in a storage system [Chang and Roberts 1979], proposer and acceptors in Paxos [Lamport 2001]). As a result, treating distributed system configurations and transitions monolithically, as is done in existing techniques, may still lead to redundant computations due to transitions that “overlap,” i.e., share computations that update the same (or partially same) local state (defined in §4.1). We improve upon existing techniques and present an efficient symbolic explorer that recognizes and exploits redundancies not only in configurations (states) and interleavings, but also in the computations in overlapping transitions.

**Our Approach.** To ground our contributions in a real-world setting, we consider distributed systems modeled with P [Desai et al. 2013a, 2018], a state-machine-based programming language for modeling and specifying distributed systems. P is being used across industry and academia for analysis of complex distributed systems [Desai 2022; GitHub 2021]; e.g., to reason about Amazon S3’s core distributed protocols [GitHub 2021] and the USB device driver stack that shipped with Microsoft Windows 8 [Desai et al. 2013a]. Teams across Amazon are using P to reason about the core distributed protocols driving their services. We present our approach as a symbolic explorer for P programs, though our ideas apply to systematic exploration of distributed systems in general.

As a first step for efficient systematic exploration of P programs, we adapt macro-step semantics for actor systems [Agha et al. 1997], on top of which we design a novel symbolic stateful explorer. To identify redundancies in transitions on-the-fly, we propose a novel, fine-grained symbolic representation of sets of configurations. Our symbolic representation is inspired by *value summaries* (sets of guard-value pairs) [Sen et al. 2015]. We adapt value summaries by introducing *Schedule-Control-Input guards (SCI Guards)* to capture symbolic scheduling choices in addition to control and input nondeterminism. We also propose *composite* value summaries and symbolic operations on them to get fine-grained representations, which we show are *canonical*. Canonicity helps our explorer identify redundant configurations as well as overlapping transitions.

We lift the macro-step P semantics to operate over these symbolic representations, where a single *symbolic transition* in the lifted semantics can capture *multiple overlapping transitions* in the original semantics, avoiding redundant computations in these transitions. We design our explorer so that it includes an interface compatible with abstractions for state-space reduction and with partial-order and other reductions for avoiding redundant interleavings. To avoid redundant interleavings, our explorer supports a persistent-set based partial-order reduction [Clarke et al. 2001; Peled 2018] that we adapt to P. We provide theoretical guarantees for the soundness and efficiency of our explorer.

**P5ym.** We implemented our approach in P5ym, a symbolic explorer for P programs. We compare P5ym with the state-of-the-art stateful explorer TLC (model checker for TLA+ [Lamport 2002; Yu et al. 1999]) and show that P5ym outperforms TLC on many open source distributed system benchmarks, finishing verification when TLC times out on two benchmarks, and achieving a runtime improvement of 2.5X over TLC on average (geometric mean) on the remaining benchmarks. P5ym can also successfully verify common distributed protocols and challenging industrial case-studies of four complex real-world distributed protocols used at Amazon Web Services (AWS) where other verification tools failed.
Contributions. In summary, we make the following main contributions:
- We present a novel, fine-grained symbolic representation of configurations in a distributed system (§4) that helps avoid redundancies during exploration. Inspired by MultiSE, we extend canonical value summaries to include schedule nondeterminism and represent common composite data structures (e.g., tuples, lists, maps) in a fine-grained, decomposed manner. To the best of our knowledge, our work is the first to use value summaries for distributed systems. These representations are not specific to P and could be used by other distributed systems frameworks.
- We propose a new symbolic stateful explorer for P programs (§5), where the macro-step semantics (§3) is lifted to leverage our novel value summaries for efficient exploration of reachable configurations while avoiding redundancies in configurations, interleavings, and overlapping transitions. Additionally, it can use abstractions (§6.2) to handle infinite-state systems.
- We implemented our ideas in a prototype tool PSYM that includes an extensible filter interface (§6.2) for integrating POR and other reductions with our symbolic explorer.
- We demonstrate PSYM’s efficacy on real-world benchmarks, including industrial scale distributed protocols (§7). (An extended case study with abstractions and additional tool comparisons are described in an extended version of the paper [Pick et al. 2023b].)

2 MOTIVATING EXAMPLE AND KEY IDEAS

We first introduce P and then highlight the key ideas of our approach on a motivating example.

2.1 P Language

P is a state-machine-based programming language for modeling and specifying complex distributed systems. P was used to implement and validate the USB device driver stack that ships with Microsoft Windows 8 and Windows Phone [Desai et al. 2013a] and is used extensively in Amazon (AWS) for formal modeling and analysis of complex distributed systems such as the core distributed protocols involved in Amazon S3’s strong consistency launch [Desai 2022; Desai et al. 2021; GitHub 2021]. P currently leverages randomized stateless exploration [Desai et al. 2015; Microsoft Coyote 2022] to find critical bugs in industrial-scale distributed protocols. Randomized search (run on a distributed cluster) is highly effective in finding low-probability bugs but fails to provide correctness guarantees. Hence, there is a need for a verification backend for P that can handle industrial-scale systems.

![Figure 1. An example P program (adapted from the TransDPOR paper [Tasharofi et al. 2012]).](image)

2.2 Motivating Example in P

A P program is a collection of concurrently executing state machines that communicate with each other by sending messages (i.e., events and payloads) asynchronously. (The underlying model of computation is similar to actors [Agha 1986].) Fig. 1 presents a simple P program. It consists of three types of state machines: Server (line 1), Worker (line 21), and Registry (line 36). The Server creates a set of Workers and sends them work items to be processed, and the Registry maintains
the set of all workers in the system. State machines in P communicate by sending events with payloads. Line 19 declares the eWorkItem event that has an associated payload of machine reference type. Each machine has a start state (e.g., line 3), where it starts execution after being created. Each state has an entry handler that is executed upon entering that state. The entry handler for the Ini state of Server creates one Registry machine (line 6) and three Worker machines (line 9), and then sends each Worker a work item to be processed (line 11) along with a reference to the Registry machine. Each machine can also have an associated set of local variables (e.g., Registry has a local variable workerIds). After executing an entry procedure, a machine blocks to receive an event. On receiving an event, the event’s handler is executed, transitioning the system from one configuration (global state) to another. If there are no messages in its buffer, a machine blocks until a message is received. For example, the Registry machine, after entering the Ini state, blocks to receive an event. On receiving the eRegisterWorker event, it executes the corresponding event handler (line 41) that adds the id in the payload to the local set. The Server, Registry, and three Worker machines execute concurrently, asynchronously sending messages to each other. In our example, the program’s initial configuration is one in which one Server instance has been created.

2.3 Systematic Exploration: Baseline Macro-step Semantics and POR for P

As a first step for efficient systematic exploration of P programs, we adapt macro-step semantics for P (§3). Macro-step semantics have been demonstrated to drastically reduce the number of interleavings for sound verification in actor systems [Sen and Agha 2006a]. The basic idea is to treat transitions starting from (and including) a receive of a message at an actor, up until (but not including) the next receive of a message at that actor, as a single atomic step. Fig. 2 shows the execution tree for the motivating example under macro-step semantics. Each node denotes a configuration (i.e., a global state) of the program, and each branch corresponds to an event received by a machine in the system. (For simplicity of exposition, we omit branches corresponding to dynamic machine creation though we handle this in Psym.) We have labeled the receives of each event (see lines 28 and 40 in Fig. 1): w0, w1, w2 represent the receive of eWorkItem events at the Worker machine instances with ids 0, 1, 2, respectively. Similarly, r0, r1, r2 represent the receive of eRegisterWorker at the Registry machine instance with payloads 0, 1, 2, respectively.

A naive systematic explorer would explore the entire execution tree shown in Fig. 2, which is already based on macro-step semantics. We improve upon this by applying a persistent-set-based POR technique for P programs called Pred (§6.2). When applying Pred to our example, the parts in red are found to be redundant and not explored. For real-world distributed systems, even with POR, there remain redundancies during exploration that present a challenge for scalability.

2.4 Symbolic Stateful Exploration using Value Summaries

Popular explicit-state model checkers (e.g., SPIN [Holzmann 1997], TLC [Yu et al. 1999]) leverage state caching to avoid exploring revisited states, along with applying POR to avoid exploring redundant interleavings. In Fig. 2, all blue subtrees need not be explored if state caching is used.
example, $c_2$ is the same configuration as $c_1$ and will not be explored if $c_1$ is visited first. However, such approaches will separately compute the orange transitions in Fig. 2. Because these transitions arise from two different configurations, traditional explicit-state explorers will perform the computations separately for each configuration. In this work, we aim to eliminate such redundant computations.

**Symbolic State Exploration.** Our approach is based on symbolic representations of configurations. Similar to well-known symbolic exploration algorithms [Chaki and Gurfinkel 2018], we compute a set of frontier configurations starting from an initial configuration, where each symbolic step considers possibly multiple transitions from the frontier set to compute the set of next configurations. The novelty in our work is in our new symbolic representations of configurations of distributed systems (§4) and in our explorer’s ability to avoid redundant computations in overlapping transitions (§4.1, §5).

**Putting It All Together.** The blue and orange parts of Fig. 2 show the additional work saved by using our symbolic stateful exploration on top of using macro-step semantics and POR. To the best of our knowledge, our work is the first to target redundancies in overlapping transitions, and the first to use value summaries for distributed systems exploration. Because distributed systems often involve having several instances of processes that exhibit the same or similar behaviors, we expect a high number of transitions to have shared behaviors that we can handle efficiently.

**Beyond systematic exploration.** The ability to perform scalable stateful exploration is a useful utility. Beyond systematic exploration, it also allows discovery of invariants through computation of fixed points (§5.3) and the use of abstractions (§6.1). Discovering invariants on small instances of distributed protocols is a crucial component in some recent approaches such as I4 [Ma et al. 2019] and DistAI [Yao et al. 2021] that generalize results from small instances to large or arbitrary-sized instances. Our symbolic explorer could be integrated with such techniques in future work.

## 3 ADAPTING MACRO-STEP SEMANTICS FOR P

We now describe an adaptation of macro-step semantics for P, which provides a baseline for our symbolic explorer. This semantics treats transitions starting from and including a receive of a message at a state machine up until but not including the next receive of a message at that machine as a single atomic step, i.e., only receives of messages at different machines are interleaved. As in actor systems [Agha et al. 1997], the interleaving semantics is equivalent to its macro-step semantics. All executions of a P program under interleaving semantics (as seen in prior work [Desai et al. 2014, 2013a, 2018, 2015]) also have a Mazurkiewicz-equivalent execution in the macro-step semantics.

### 3.1 Notation for P Semantics

**Machine.** Let $\mathcal{A}$ represent the set of names of all machine types. Let $I$ represent the set of all the machine identifiers referencing dynamic instances of machine types in $\mathcal{A}$. Let $S_{id}$ represent the set of local states for a machine with reference identifier $id \in I$.

**Message.** Let $E$ be the set of names of events and $V$ be the set of all possible payload values that may accompany any event. Let $M$ represent the set of all possible messages. Each message is a tuple $(src, ev, v, tgt, mid) \in (I \times E \times V \times I \times N)$ where src is the source or sender of the message, ev is the event being sent, v is the associated payload value with the event, tgt is the target or intended recipient of the message, and mid is an unique identifier associated with each message. For any $m \in M$, we refer to its components as $m.src$, $m.ev$, $m.v$, $m.tgt$, and $m.mid$ respectively.

**Event Handlers.** Recall that each state machine declaration in P has a set of event handlers per state. Each event handler is responsible for processing messages with the associated event type (e.g., in Fig. 1, line 26 defines an event handler for messages with event type $\text{eWork}!\text{ctx}$). All messages with the same event type that are received in the same target machine state thus have the same event handler. Let $H$ be the set of all event handlers in the given P program. Let $H : S_{id} \times E \rightarrow H$ be a partial function that given the current state of a machine and an event type maps it to the
3.2 Macro-step Semantics for P

A P program steps from one configuration to another via a labeled-transition, each of which receives a message. A macro step (or schedule step) is represented as \( c \xrightarrow{m} c' \), where label \( m \) indicates the message received during the transition that takes the system from configuration \( c \) to \( c' \) by executing the appropriate event handler. The rules for macro-step semantics are shown in Fig. 3.

**Schedule Step.** The first rule presents the macro-step transition, given a mapping \( H \) from machine states and messages to event handlers. A step \( c \xrightarrow{m} c' \) is only possible whenever the transition that receives message \( m \) is enabled in \( c \). Formally, this condition (labeled (c1)) is denoted by predicate \( \text{enabled}(c, m) \) defined as: \( m \in B_c \land \forall m' \in B_c, m' \neq m \land m'.tgt = m.tgt \Rightarrow (m', m) \notin SO_c \).

In P programs, because of FIFO buffer semantics, messages must be received in an order consistent with the causal relation of their sendings. In particular, (1) any message \( m' \) sent in response to a receive of another message \( m \) must be received after \( m \) is received, and (2) any two messages sent to the same target must be received in the order they were sent, respecting the send-order relation. The first condition is handled by adding messages to \( B_c \) immediately upon sending them and removing them immediately upon receiving them, as noted in Fig. 3. The second condition is addressed by the \( \text{enabled}(c, m) \) predicate for the P programs, which ensures that for any message \( m \) that can be received by \( m.tgt \), all other messages \( m' \) with the same target are sent after \( m \).

Executing the macro step involves removing the message \( m \) from buffer \( B_c \) (c2), and executing the corresponding event handler \( h \) in the target machine (c3) on message \( m \). The condition (c4) represents execution of the sequence of statements \( h(m) \) at the target machine \( m.tgt \), which may
only change the local state of the target machine and the global buffer (if a send statement is executed). \( \rightarrow^* \) denotes the reflexive transitive closure of \( \rightarrow \) transitions (used for statements).

**Statements.** In Fig. 3, the next five rules present semantics of some of the statements in P handlers that have non-trivial semantics when lifted for symbolic exploration (revisited in §5.1). The Sequence, Assign-Var, If-Else, and If-Then rules are straightforward and present the common semantics for sequence, assignment, and if-then-else statements, respectively. Here, \( \perp, \top \) denotes the big-step semantics for evaluation of an expression \( e \) to a value \( v \), and \( \top, \bot \) denote the Boolean values true, false, respectively. The Send rule present the semantics of asynchronous send statement send \( t, ev, v \), which sends a message \( m \) (c5) with event \( ev \), payload \( v \) to target machine \( t \), and a fresh message id. It adds message \( m \) to the global buffer (c6) and updates the send-order relation (c7) so that all previously-sent messages in the buffer that came from the same source machine are related to the new message \( m \). SO\(_x\) is updated so if \( B_x\) is nonempty, there is at least one message \( m \) for which enabled\((c, m)\) holds. Finally, the Choose rule presents the semantics of the choose operation in P used for introducing data non-determinism in the programs.

**Execution.** An execution of a P program is a sequence of macro steps for it \( c_0 \xrightarrow{m_0} \cdots \xrightarrow{m_{n-1}} c_n \xrightarrow{m_n} \cdots \). They are partial (vs. full) when they are finite and the last configuration can take a macro step.

4 SYMBOLIC REPRESENTATION OF CONFIGURATIONS: VALUE SUMMARIES

In this section, we describe details of our proposed symbolic representation of configurations for distributed systems. These are critical for efficient symbolic exploration (§5) and could be potentially useful in other verification techniques for distributed systems. The technical definitions are summarized in a cheat-sheet shown in Fig. 4, with explanations in the related subsections.

### 4.1 Identifying Overlapping Transitions

Recall from our motivating example (§2.4) that a novel element of our approach is to identify redundancies due to overlapping transitions, e.g., the transitions from \( c_4 \) in Fig. 2. A transition corresponds to a schedule step in the concrete semantics (Fig 3), that receives a message at a target machine, executes the corresponding event handler, and moves the system to the next configuration. Fig. 4(a) shows the technical definition for overlapping transitions. Informally, two transitions overlap if they operate on the same machine \( (tgt) \), and at least one of their update computations can be performed by the same function \( (h) \), on the same local variables \( (S_v) \) and payloads \( (o_v) \). Other update computations may also be performed, constituting the transitions’ non-overlapping parts.

Identifying functionally equivalent computations in arbitrary transitions requires additional semantic analyses in general. In this work, we instead use a simple syntactic approach, using having the same event handler as a proxy: when two transitions execute the same event handler at the same target machine, we aim to ensure that their overlapping computations are executed only once.

### 4.2 Requirements on Symbolic Representations of Configurations

We now consider requirements on our symbolic representations that will help identify overlapping transitions on-the-fly during symbolic exploration and achieve savings by avoiding redundancies. We illustrate them by revisiting the motivating example in Fig. 2.

**R1. Fine-grained Component-level Representation of Configurations and Messages.** We require a decomposed representation of configurations, which maintains a separate representation for each configuration component, so that we can identify when transitions correspond to the same event handler. Recall that in P, event handlers are identified by the local state and event type; furthermore, the possible event types and payloads for the next transition are determined by the global message buffer. Thus, the decomposed representation should be fine-grained enough to represent the local state of a machine instance, and the event types and payloads of messages.

(a) Overlapping transitions

Let \( c_1 \xrightarrow{m_1} c_1', c_2 \xrightarrow{m_2} c_2' \) be transitions and \( tgt, S_o, v_o \) be such that the following conditions hold:
- Same target, \( tgt \): \( tgt = m_1.tgt = m_2.tgt \)
- Local state overlap, \( S_o \): \( \forall x. S_o[x] = c_1.L[tgt].x \iff c_1.L[tgt].x = c_2.L[tgt].x \)
- Payload overlap, \( v_o \): \( v_o[p] = m_1.v.p \iff m_1.v.p = m_2.v.p \)

For \( i \in \{1,2\} \), let \( \{x_1, \ldots, x_n\} \) be the domain of local variables \( c_i.L[tgt] \), and let \( \{f_1(x_1), \ldots, f_n(x_n)\} \) be the update functions that compute updates to these local variables, i.e., for \( 1 \leq j \leq n \), \( c_i.L[tgt][x_j] = f_i(x_j)(c_i.L[tgt], m_i.v) \).

The transitions \( c_1 \) and \( c_2 \) are overlapping if there exists an \( x \) in the domain of \( S_o \), such that there exists a function \( h \) with \( h(S_o, v_o) = f_i(x)(c_i.L[tgt], m_i.v) = f_j(x)(c_j.L[tgt], m_j.v) \), and there exists \( i \in \{1,2\} \) such that \( h(S_o, v_o) \neq c_i.L[tgt][x] \) (i.e., \( x \) is changed by \( h \) in \( c_1 \) or \( c_2 \)).

(b) Primitive value summaries: Invariant properties for guarded values \((g, v)\)

- Non-overlapping guard \( \forall (g', v') \in pvs.v \neq v' \Rightarrow g \land g' \Rightarrow \bot \)
- Unique value \( \forall (g', v') \in pvs.v \neq g' \Rightarrow v \neq v' \)
- Non-vacuous \( g \Rightarrow \top \)

(c) Composite value summary representations

- Tuple value summary \((x_0, \ldots, x_n)\) where all \( x_i \) are value summaries with same domain
- List value summary \((s, ls)\) with integer value summary \( s \), value summary list \( ls \)
- Map value summary \( mp : \tau \rightarrow \tau' \) where every element in \( \tau' \) is a value summary

(d) Core operations on primitive/composite value summaries

Removing spurious guards: \( rmf(pvs) = \{(g, v) | (g, v) \in pvs \land g \neq \bot\} \)

Domain operation: \( D(vs) = \)
- \( \forall \{D(vs_i) | i \in \{0, \ldots, n\}\} vs = (vs_0, \ldots, vs_n) \)
- \( \forall \{D(vs') | \exists k. vs[k] = vs'\} vs \) is a map

Merging operation: \( M((vs_0, \ldots, vs_k)) = \) if \( k = 1 \) then \( M(vs_0, vs_1) \) else \( M(vs_0, M(vs_1, \ldots, vs_k)) \)

\( M(vs_0, vs_1) = \)
- \( vs_0 \cup vs_1 \)
- \( (M(vs_0^0, vs_1^0), \ldots, M(vs_0^n, vs_1^n)) vs_i = (vs_0^i, \ldots, vs_1^n) \) (tuple or list value summary)
- \( \forall j \neq i, j, i \in \{0,1\} vs_j = [] \)
- \( M(x_0, x_1) :: M(xs_0, xs_1) vs_0 = x_0 :: vs_0 \land vs_1 = x_1 :: vs_1 \) where :: is cons
- \( mergeMap(mp_0, mp_1) vs \) is a map

where \( D(vs_0) \land D(vs_1) = \bot \) (i.e., non-overlapping domains) and \( mergeMap(mp_0, mp_1) \) is

\( \{k \mapsto vs | vs = M(mp_0[k], mp_1[k]) \vee \exists i, j \in \{0,1\}. vs = mp_i[k] \land k \notin keys(mp_j)\} \)

Restriction operation: \( (vs|\phi) = \)
- \( \{r mf((\{(g, v, \phi) | \exists (g, v) \in vs\}) vs \) is primitive \)
- \( \{(vs_0|\phi), \ldots, (vs_n|\phi)\) vs = \( (vs_0, \ldots, vs_n) \)
- \( \{(s|\phi), (ls|\phi)\) vs = \( (s, ls) \)
- \( \{k \mapsto (vs'|\phi) | vs[k] = vs'\) vs is a map

Getting guards for values: \( \exists g. (g, v) \in pvs \Rightarrow getGuardFor(pvs, v) = g \)

Updating under a guard: \( updateUnderG(vs, vs', g) = M((vs|\neg g), (vs'|g)) \)

Fig. 4. Technical cheat sheet for overlapping transitions and value summaries.

separately. For example, \( c_4 \) in Fig. 2 has messages for \( r_0, r_1, r_2 \) in its global buffer. These messages all have event type \( @RegisterWorker \) and are all sent to the Registry instance – this can be detected only with a fine-grained representation of configurations and message components.
**R2. Canonicity of Symbolic Representations.** We need to identify when configuration and message components are equivalent. Canonical representations – i.e., where equivalent sets of configurations/messages have identical representations – make such identification quick and cheap, and allows us to efficiently detect when transitions have the same event handlers. We also require that each configuration update maintain canonicity, preventing the exploration from revisiting redundant configurations at the same execution depth. For example, in Fig. 2, canonicity ensures that $c_1$ and $c_2$ have the same representation during exploration, making it easy to detect and avoid redundant exploration of the blue subtree under $c_2$. (Note that POR techniques cannot detect the equivalence of $c_1$ and $c_2$ because they occur under different interleavings of dependent transitions.)

**R3. Association of Nondeterministic Choices with Values.** Finally, we require that symbolic representations implicitly implement a function from sequences of nondeterministic choices taken during exploration to specific values, in order to understand the explored behaviors and to produce counterexample traces. For systematic exploration, these sequences should include all scheduling choices taken during execution, as well as any choices due to control-flow or data nondeterminism.

### 4.3 Value Summaries for Configurations of Distributed Systems

To satisfy requirements **R1-R3**, we use a symbolic representation inspired by MultiSE [Sen et al. 2015], a systematic exploration framework for sequential programs. In MultiSE, value summaries represent sets of concrete values for individual variables under different guards, where guards capture control-flow and input nondeterminism. Importantly, value summaries provide a canonical representation of the program state, which other symbolic representations, such as those based on formulas in solvers for Satisfiability Modulo Theories [Barrett et al. 2009] do not. In this work, we extend guards in value summaries to capture scheduling choices also. We also propose composite value summaries to represent components of a distributed system in a fine-grained manner.

#### 4.3.1 Guards and Scheduling Choices

Value summaries are based on guard-value pairs, called guarded values, where a guard is a propositional logic formula over Boolean-valued guard variables, and all guards in a summary are non-overlapping [Sen et al. 2015]. We propose the use of SCI Guards, which represent choices in the presence of Scheduling, Control, and Input nondeterminism.

As we will see in §5.1, we introduce fresh variables in guard formulas to symbolically encode the set of possible choices for any nondeterministic choice made during exploration. This associates a unique guard with each possible nondeterministic choice, leading to unique guards for each sequence of choices arising during exploration. Thus, this representation satisfies **R3**.

#### 4.3.2 Primitive Value Summaries

A primitive value summary **pvs** is a set of guarded values. Each pair $(g, v)$ represents a fact that the value $v$ is taken under the guard $g$. We define three invariant properties for each guarded value $(g, v)$ in a value summary **pvs**, shown in Fig. 4(b). Two value summaries are equivalent (denoted with $\equiv$) iff they have equivalent values under all the guards.

**Tunable Value Summaries.** Guards that capture nondeterministic choices are represented symbolically; however, the representation for the value components of each guarded value, as in MultiSE [Sen et al. 2015], is not restricted, as long as it supports a check for equivalence. (Though for best performance, canonical representations should be used.) In our work, we use concrete as well as abstract values. We assume concrete values when discussing our symbolic explorer (§5), to focus on the symbolic guard-based operations needed for efficient exploration, but discuss how to use a simple predicate abstraction [Graf and Saïdi 1997] to represent values in §6. Thus, our value summaries are tunable, where the value representation can range from concrete to abstract.

#### 4.3.3 Composite Value Summaries

We use primitive value summaries as building blocks to propose composite value summaries for representing composite data structures (e.g., tuples, lists). Composite value summaries are also a novel contribution of our work. They enable fine-grained symbolic
representations of configurations and messages in distributed systems, satisfying requirement R1. A composite value summary is constructed out of other value summaries (primitive or composite). We show them for tuples, lists, and maps in Fig. 4(c) – these are needed to represent P programs.

Example 4.1. Consider the messages \((\text{worker}_i, e\text{Register}\text{Worker}, i, \text{reg}), i \in \{0, 1, 2\}\) sent by \text{Worker} machines (Fig. 1). The following value summary captures these under distinct guards where \(g_0, g_1, g_2\) are formulas with pairwise empty conjunctions, and \(id_0, id_1, id_2\) are unique message ids:

\[
\{(\text{worker}_i, e\text{Register}\text{Worker}) \mid i \in \{0, 1, 2\}\}, \{(\top, e\text{Register}\text{Worker})\}, \{(g_i, i) \mid i \in \{0, 1, 2\}\}, \{(\top, \text{reg})\}, \{(g_i, id_i) \mid i \in \{0, 1, 2\}\}
\]

Note the lack of redundancy in representing the three messages’ types – \(e\text{Register}\text{Worker}\) occurs only once in the value summary.

4.3.4 Core Operations on Value Summaries. We now discuss core operations on value summaries, shown in Fig. 4(d), that will be used for symbolic exploration. In particular, the lifted semantics (§5.1) used by the exploration algorithm in §5 uses the \textit{domain} operation to get the guard under which a configuration should be modified, and the \textit{merge} and \textit{restrict} operation to update only the part of the configuration that should be modified by a transition.

\textbf{Domain.} The domain \(D(vs)\) of a value summary \(vs\) (symbolically) is a Boolean formula that represents the set of guards under which it has a value. In particular, note that the domain of a list value summary \((s, ks)\) is recovered by taking the domain of its size component \(s\). Updates to a list value summary must maintain the invariant that each element of \(ks\) has a domain that implies \(D(s)\).

\textbf{Merging.} The result of merging two value summaries \(vs_0, vs_1\) another value summary, defined under the condition \(D(vs_0) \land D(vs_1) = \bot\) (i.e., non-overlapping domains). The result of merging a set of value summaries is defined similarly.

\textbf{Restriction.} Restricting a value summary \(vs\)’s domain by a guard \(\phi\), denoted \((vs|\phi)\), yields a value summary with domain \(D(vs) \land \phi\), which represents only the values in \(vs\) under guard \(\phi\).

\textbf{Getting Guards for Values.} The function \textit{getGuardFor} takes a primitive value summary \(pvs\) and a value \(v\) and returns an SCI Guard \(g\) such that \((g, v) \in pvs\).

\textbf{Updating under a Guard.} Updating a value summary \(vs\) under an SCI Guard \(g\) allows us to model assignments under the nondeterministic choices captured by \(g\). Such an update can be easily defined by using merging \((M)\): \textit{updateUnderG}(vs, vs', g) = M((vs|\neg g), (vs'|g)). Note that the restriction operations make the domains of the arguments to the merge non-overlapping.

4.4 Canonicity of Value Summaries

We now state an important theorem for value summaries that is maintained by the operations on value summaries defined above (a proof is provided in the extended version [Pick et al. 2023b]).

\textbf{Theorem 4.2 (Canonicity of Value Summaries).} If a canonical representation is used for propositional formulas and for value components of guarded values, then all value summaries are such that, for any two value summaries \(vs_0, vs_1\), \(vs_0 \equiv vs_1\) iff \(vs_0 = vs_1\).

Here \(\equiv\) denotes semantic equivalence and \(=\) denotes syntactic equality. By this theorem, value summary representations satisfy requirement R2. Like MultiSE [Sen et al. 2015], we represent guards with Binary Decision Diagrams (BDDs) [Bryant 1986]. BDDs are used in a very limited way here – only to represent the guards in value summaries. In general, the size of state representations is a scalability concern, but the regular structure in practical programs of interest can help avoid blowup. Our tool successfully handles many standard and industrial benchmark programs (see §7).

5 SYMBOLIC STATEFUL EXPLOSION USING VALUE SUMMARIES

In this section, we first describe how to lift P values and semantics to value summaries, and then present our novel symbolic explorer that operates over the lifted P program.
5.1 Lifting Values and Functions to Value Summaries

Let $VS(r)$ represent the value summary representation for any value of type $r$. Let $\ell : r \rightarrow VS(r)$ denote the lifting function\(^1\) and $Vals$ denote the function that determines the set of values represented by a lifted value summary under any guard. $Vals$ and $\ell$ meet the following requirements:

- For any $a$, $Vals(\ell(a)) = \{a\}$
- For any function $f$, $\ell(f)$ is deterministic and $Vals(\ell(f)(\ell(\text{args}))) = \{r \mid f(\text{args}) \text{ can return } r\}$

Functions $\ell$ and $Vals$ can be seen as restricted forms of an abstraction and concretization function, respectively, where abstraction does not introduce any imprecision.

**Lifting Data Structures.** The P language contains several types that may be used in event and state handlers: primitive types, tuples, lists, sets, and maps [P-GitHub 2023]. For these types other than sets, we directly use their corresponding value summaries to represent them; for sets we use list value summaries, e.g., by representing sets as sorted lists with no duplicate elements. Formally, we have $\ell(p) = ((\top, p))$ for primitive value $p$ and $Vals(pvs) = \{v \mid \exists g . (g, v) \in pvs\}$ for primitive value summary $pvs$. Fig. 5 shows how to lift concrete composite data structures by applying $\ell$. 

**Lifting Functions.** A function $f : r \rightarrow r'$ over primitive types can be lifted to $\ell(f) : VS(r) \rightarrow VS(r')$, which iterates over values in the provided primitive value summary $vs$ and combines guarded values with the same values into a single guarded value as follows: $\lambda vs . \{((\forall G, v') | G = \{g | (g, v) \in vs \land f(v) = v'\}\}$. In effect, evaluating $f(x)$ requires evaluating $f$ on each concrete value represented by $x$. For composite value summaries, lifting is more involved, since for efficiency, we want to avoid having to enumerate all the concrete values represented by a composite value summary. As an example, the equality check $v_0 = v_1$, which returns a value $b \in \{\top, \bot\}$, can be lifted to $=_{\ell}$, which takes two value summaries and returns a Boolean value summary $VS(\{\top, \bot\})$. For composite arguments, such a lifting of $=_{\ell}$ should recursively evaluate $=_{\ell}$ on corresponding components of the value summaries and return the conjunction of the results.

**Example 5.1.** Consider checking the equality of the following tuple value summaries, where $X, Y$ are finite subsets of $\mathbb{N}$: $\{(g_i, x_i) \mid i \in X\}, \{(g_j, y_j) \mid j \in Y\}$. We first recursively check equality of the first elements, yielding result $\{(g_0, \top), (\neg g_0, \bot)\}$. This check involves $|X|$ comparisons, comparing each $x_i$ with $x_0$. We then do the same for the second elements, yielding the same in $|Y|$ comparisons. The equality check gives result $\{(g_0, \top), (\neg g_0, \bot)\}$. Note that if we used a primitive value summary representation of these tuples rather than composite value summaries, we would need to perform at least $|X| \times |Y|$ comparisons, since we would have to compare each pair in the first summary to each in the second.

When implementing lifted data structures and functions, it is important for the sake of efficiency to expose and take advantage of the decomposed representation of composite value summaries as much as possible. For example, the list value summary representation maintains and exposes the size of the list, which allows efficient implementations of operations to get the value summary’s domain or the list’s size. The extended version contains more details about lifting [Pick et al. 2023b].

5.2 Lifted Semantics for P Programs

Fig. 6 shows the result of lifting the rules for P semantics from Fig. 3. See the extended version for the full set of rules [Pick et al. 2023b]. While we use the same notation for configurations, messages,

---

\(^1\) We overload the notation $\ell(\cdot)$ to denote lifting of both values and functions.


\[
\text{\textbf{Schedule Step}}
\]
\[
\text{choices} = (\lambda x. \{ y \mid \text{enabled}(y, x)\})(c) \\
g = \phi \land \left( \bigvee \{ D(\text{choice}) \mid \text{choice} \in \text{choices} \} \right) \quad g, \{ \} \vdash \text{choose choices} \downarrow m
\]
\[
c' = (\lambda x. B_c \setminus \{ o \}, \text{SO}_c)[\theta_0, \ldots, \theta_{n_m} = \{ o \}, \text{m}(m, m_0), \text{m}_0, \text{m}_1) = b_0
\]
\[
V_0 \leq i \leq n_m = \{ o \}, \text{m}(m, m_0), \text{m}_0, \text{m}_1 \}
\]
\[
\text{g}_0, \text{m}_0, \text{m}_1 \vdash (h_0(\text{m}_0), c'_0) \rightarrow^* (\text{skip}, c'_0) \ldots (\text{g}_0, \text{m}_0, \text{m}_1 \vdash (h_0(\text{m}_0), c'_0) \rightarrow^* (\text{skip}, c'_0)) \rightarrow^* (\text{skip}, c'_0)
\]
\[
H, \phi + c \Rightarrow c'
\]

\[
\text{\textbf{Assign-Var}}
\]
\[
c' = (\lambda x. \{ \text{id} \mid \text{updateUnderG}(x, a, g) \}, B_c, \text{SO}_c)
\]
\[
g, \text{id} \vdash \text{(x := e, c)} \rightarrow (\text{skip, c'})
\]

\[
\text{\textbf{Send}}
\]
\[
m = \{ (\text{id, e, v, o, t, id, \text{fresh}(\text{id}, c))(g) \}
\]
\[
\text{SO}_c = \text{SO}_c \cup \{ (\lambda x. y. \{ (m', x) \mid m'. \text{src} = x, \text{id} \land m' \in y \}) (m, (B_c, g)) \}
\]
\[
g, \text{id} \vdash \text{(send (id, e, v, o, c)} \rightarrow (\text{skip, c'})
\]

\[
\text{\textbf{If}}
\]
\[
g_0 = g \land \text{getGuardFor}(a, T)
\]
\[
g_1, \text{id} \vdash (S_0, c) \rightarrow^* (\text{skip, c'})
\]
\[
g, \text{id} \vdash \text{(if e then S} \_0 \text{else S} \_1 \text{c} \rightarrow (\text{skip, c'})}
\]

\[
\text{\textbf{Choose}}
\]
\[
B = \text{getFreshGuardVariables([log([V])])}
\]
\[
V = \{ u_1, \ldots, u_V \}
\]
\[
u = M((\{ u \mid \text{encodeUsing}(i, B, \text{V}))\}_{i \in [V]})
\]
\[
g, L_c, \text{id} \vdash \text{choose V} \downarrow \{ (a | v) \}
\]

Fig. 6. Lifted semantics for P programs where \( \setminus \) and \( \cup \) are lifted versions of their corresponding operations.

handlers, etc., note that these have all been lifted to value summaries in Fig. 6. In particular, the set of pending messages \( B_c \) and the send order \( \text{SO}_c \) are also value summaries – this enables processing overlapping transitions together when they have the same event handler. The map \( H \) has also been lifted to map value summary arguments to a lifted handler. Because these have been lifted to value summaries, we meet requirements R1-R3 (§4.2) for our configuration and message representations.

**Guard in a Context.** For all rules, there is a guard \( g \) (or \( \phi \) in \text{Schedule Step}) in the context – this keeps track of the current SCI Guard in the execution so far, and all lifted operations occur under \( g \) (or \( \phi \)). Recall that the SCI Guard characterizes all the nondeterministic choices made in the execution, thus \( g \) determines a subset of the paths in the execution tree and restricts which configurations in the front-end get updated. More formally, if \( g, \text{id} \vdash (S, c) \rightarrow (S', c') \), we have that \( (c|\neg g) = (c'|\neg g) \) (and analogously for the \( \Rightarrow \) step). This can be seen in the \text{Send} rule’s use of restriction and in the \text{Assign-Var} rule’s use of \text{updateUnderG} (both defined in Fig. 4(d)). Similarly, the operations also occur under the domain of the current configuration \( D(c) \), so that if \( g, \text{id} \vdash (S, c) \rightarrow (S', c') \), we have that \( D(c') \) implies \( D(c) \) (and analogously for the \( \Rightarrow \) step).

**Lifted Event Handler.** For any configuration, all enabled messages that share the same event handler are handled by a single invocation of a lifted version of the event handler, which is executed on a value summary that captures all their payloads. This results in fewer handler invocations and less redundant computation than handling them separately. As we will show in §7.2, there are many opportunities in distributed systems to handle messages together this way.

**Choose.** The \text{Choose} procedure implements the semantics for the \text{choose} operation, which takes a set \( V \) of value summaries. It generates a single value summary that captures all the values \( v_i \in V \). Specifically, it introduces fresh Boolean guard variables \( B \) \( (c^{19}) \) to construct a primitive value summary \( v \) \( (c^{19}) \), such that \( \text{Vals}(v) = V \) and each guarded value in \( v \) uses only variables in \( B \). This effectively encodes all the possible nondeterministic choices in \( V \). In particular, the function \( \text{encodeUsing}(i, B, \text{size}) \) is defined for \( 1 \leq i \leq \text{size} \) and behaves as follows: for \( i < \text{size} \), it gives the binary encoding \( \text{bin}(i - 1, B) \) for \( i - 1 \) using the guard variables in \( B \), and for \( i = \text{size} \), it returns \( \bigwedge \{ \text{bin}(k - 1, B) \mid \text{size} \leq k \leq |B|^2 \} \). The effect is that \( \text{encodeUsing} \) uses \( B \) to create \( \text{size} \) symbolic partitions, each of which becomes a guard in \( v \). However, this \text{choose} operation as described leaves
some "left over" values under the guard \((\forall \{D(v_i) \mid v_i \in V\}) \land \neg D(vs))\) (when it is not false). Such values should be eliminated by restricting subsequent operations by \(D(vs)\) as leaving "left over" values can result in re-exploration of choices.

**Schedule Step.** Compared with Fig. 3, the Schedule Step rule looks the most different, since it now must advance all configurations in the frontier set rather than a single frontier configuration. It finds the guard under which there are enabled messages to schedule (c8), and then uses Choose to make a nondeterministic choice over enabled messages (c9), since scheduling nondeterminism is handled in the same way as input and control-flow nondeterminism. The enabled message value summary \(m\) here represents the set of messages that can be received next for all configurations in the frontier set. Put another way, consider the case in the original semantics where we have a set of messages \(Mess\) and configuration value \(config\), where each \(mess \in Mess\) is enabled in \(config\) (see (c1) in Fig. 3). For any configuration value summary \(c\) in the lifted semantics with \(config \in Val(c)\), if we can take a Schedule Step from \(c\), then \(Mess \subseteq Val(c)\), where \(m\) is as in (c9).

Removing \(\{m\}\) from pending messages set \(B_c\) (c10) involves removing all these messages only under their corresponding domains, introduced by the Choose operation. After this removal, which yields (intermediate) configuration \(c''\), the guards change accordingly: for any \(mess \in Val(c)\), \(mess \notin Val(c')|\neg D(m)\) where \(c'\) is \(\{B_c - D(m)\}\). "Left over" enabled messages in \(\{B_c - D(m)\}\) are eliminated in (c14).

For each distinct target and event in \(m\) under guard \(g_t\) (c11), there is a unique (lifted) event handler \(h_t\) (c12). The number of distinct targets and events in \(m\) may be less than the number of distinct messages in \(m\) – in such cases, we process these overlapping messages with the same \(h_t\). In order to process all messages, Schedule Step runs all such event handlers under each guard \(g_t\) (c13), and then finally restricts the resulting configuration to the domain \(D(m)\) under which a message was processed (c14). Note that \(m, t\) is the payload for all messages in \(m\), so each lifted handler runs only once per distinct target and event pair in \(m\), and on a value summary that represents potentially multiple message payloads. This achieves the goal in requirement R1: to identify when transitions correspond to the same event handler and handle them together.

**Assign-Var.** The Assign-Var rule is as in Fig. 3, but with the the map \(L\) lifted to be over value summaries (as in Fig. 5). The update of the map assignment to \(x\) correspondingly becomes an update of the value only under the guard \(g\), via updateUnderG.

**Send.** The Send rule is also a fairly straightforward lifting of the corresponding rule from Fig. 3. Recall that the set of pending messages \(B_e\) and the send orders \(SO_e, SO_v\) have been lifted to set value summaries. The update to the set of pending messages \(B_e\) (c16) happens only under the domain for message \(m\), which has been restricted by \(g\) (c15). The update to the send-order relation (c17) is also done similarly, adding messages only under \(g\).

**If.** The If rule replaces both of the branching rules from Fig. 3. It executes the then branch under the guard \(g_0\) for which condition \(e\) evaluates to \(\top\), and the else branch under the guard \(g_1\) for which it evaluates to \(\bot\), suitably updating the value summaries in a single step of the semantics.

**Example 5.2.** Consider running Schedule Step for the example in Fig. 1 at the root of the execution tree in Fig. 2. Suppose no POR is applied (for simple illustration), so that we must consider all interleavings of eWorkItem messages. The enabled message value summaries at this point are the following three summaries, for \(i \in \{0, 1, 2\}\) for some message ids \(id_i\):

\[
\left\{\left\langle \left(\text{T, Server}\right)\right\rangle, \left\langle \left(\text{T, eWorkItem}\right)\right\rangle, \left\langle \left(\text{T, reg}\right)\right\rangle, \left\langle \left(\text{T, worker}_i\right)\right\rangle, \left\langle \left(\text{T, id}_i\right)\right\rangle\right\}
\]

Each summary is a 5-tuple of value summaries for the message sender, event being sent, payload for the event, message target, and unique identifier of the message. For this application of Schedule Step, \(g = \top\) (c8). The resulting value summary \(m\) with all nondeterministic choices (c9) follows:

\[
\left\{\left\langle \left(\text{T, Server}\right)\right\rangle, \left\langle \left(\text{T, eWorkItem}\right)\right\rangle, \left\langle \left(\text{T, reg}\right)\right\rangle, \left\langle \left(f, worker_i\right)\right\rangle\right\}_{i=0..2}, \left\{\left\langle \left(f, id_i\right)\right\rangle\right\}_{i=0..2}\right\}
\]
Algorithm 1 Symbolic exploration of all executions.

1: procedure SymExplore(P_t)
2:   (vs_c, H, prev) ← (INITIAL_CONFIG(P_t), GET_HANDLERS(P_t), {})
3:   while bufferNotEmpty(c) do
4:     Optional: prev ← vs_c := prev
5:     vs_c ← SCHEDULE_STEP(H, notFixedPoint, vs_c)
6:     vs_c ← (vs_c | ~(checkFixed(prev, vs_c)))

Here, CHOICE has introduced fresh guard variables b_0, b_1, b_2, and guard formulas f_0 = b_0 \land b_1, f_1 = b_0 \land \neg b_1, and f_2 = \neg b_0 to encode the three possible choices. Note that there are three distinct target and event pairs in m_i (worker_i, eWorkItem) for i \in \{0, 1, 2\}, which each occur under their respective guard f_i (c11). Thus, a lifted event handler for eWorkItem (c12) for each worker is run under the guard f_i on the message payload reg (c13), yielding a value summary that symbolically represents all configurations at depth 1 in Fig. 2.

**Theorem 5.3 (Efficiency of Schedule Step).** For any program P with lifted program P_t, consider a frontier set represented by a configuration value summary c that can step to configuration c’ via the Schedule Step. Each lifted event handler in P_t is executed at most once in this step. (Proof in the extended version [Pick et al. 2023b].)

5.3 Symbolic Stateful Explorer

Alg. 1 shows our symbolic stateful explorer, which operates over a lifted program P_t and makes use of a predicate bufferNotEmpty that is true whenever the given configuration has a nonempty global buffer under some guard. It maintains a frontier set of configurations represented by a value summary vs_c, where each symbolic step performs exploration by advancing configurations along the frontier by all possible schedule steps, resulting in a breadth-first search style of exploration.

Recall from the lifted semantics of the Schedule Step, Alg. 1 (line 5) explores all scheduling choices; given H, g, c, it returns configuration c’ such that H, g ⊢ c m \Rightarrow c’ is derivable in the lifted semantics. Because Alg. 1 operates over value summaries, which are fine-grained (requirement R1) and exhibit canonicity (requirement R2), it avoids revisiting redundant configurations and retaining redundant transitions in the same step. Alg. 1 continues (line 3) until there are no enabled messages in the global buffer under any guard, or, if the check on line 4 is included, until a fixed point is found (in which case notFixedPoint = ⊥). The line 4 can be optionally included to perform a fixed point check (explained in §5.4). Without it, we assume checkFixed always returns ⊥.

**Soundness.** We now state a theorem on the correctness of our explorer in Alg. 1 (without line 4).

**Theorem 5.4 (Soundness of Symbolic Explorer).** For any program P with lifted program P_t, if there exists an execution c_0 m_0 \Rightarrow c_1 m_1 \Rightarrow \ldots of P, then Alg. 1 (without line 4) on P_t explores an execution vs_c_0 vs_m_0 \Rightarrow vs_c_1 vs_m_1 \Rightarrow \ldots, such that for all i, c_i ∈ Vals(vs_c_i) and m_i ∈ Vals(vs_m_i). (Proof in the extended version [Pick et al. 2023b].)

Intuitively, the theorem holds because each configuration in the execution of P is a part of some frontier configuration summary explored in P_t and because Schedule Step runs handlers for all the enabled messages in the frontier configurations.

**Efficient Symbolic Exploration.** Because our frontier representations meet requirements R1-R3, and our lifted semantics based on these representations are efficient (Theorem 5.3), our explorer, which executes these semantics, avoids redundancies due to identical configurations and overlapping transitions in the frontier.
We now present a precision theorem that relates executions explored by Alg. 1 on a lifted program with executions of the P program, showing that Alg. 1 does not explore any spurious behaviors. Informally, this theorem follows from the fact that Alg. 1 starts from a configuration value summary that are initial configurations of the original P program and that the SCHEDULE STEP only executes event handlers for messages that are enabled in the P program.

Consider an execution \( e = c_0 \overset{m_0}{\rightarrow} c_1 \overset{m_1}{\rightarrow} \ldots \) of a P program. We refer to \((c_0, m_0, c_1), (c_1, m_1, c_2) \ldots\) as its execution triple sequence (ets), and do so analogously for executions of lifted P programs, where the ets is a sequence of triples of value summaries. For a sequence of triples of value summaries \( \sigma_\ell = (v_{s_\ell}, v_{m_\ell}, v_{c_\ell}), (v_{s_1}, v_{m_1}, v_{c_1}) \ldots \), we let \( \text{Vals}(\sigma_\ell) \) be the set of sequences \((c_0, m_0, c_1), (c_1, m_1, c_2) \ldots \), where \((c_i, m_i, c_{i+1}) \in \text{Vals}(v_{s_i}) \times \text{Vals}(v_{m_i}) \times \text{Vals}(v_{c_{i+1}})\) and \( \wedge_1 \text{getGuardFor}(v_{s_i}, c_i) \wedge \text{getGuardFor}(v_{m_i}, m_i) \wedge \text{getGuardFor}(v_{c_{i+1}}, c_{i+1}) \) is not false. Intuitively, \( \text{Vals}(\sigma) \) is the set of sequences with triples of values that are represented by the summaries in \( \sigma \).

**Theorem 5.5 (Precision of Symbolic Explorer).** For any program \( P \) with lifted program \( P_\ell \), if Alg. 1 (without line 4) on \( P_\ell \) explores an execution \( e_\ell \), the ets \( \sigma_\ell \) of triples in \( e_\ell \) is such that for all \( \sigma \in \text{Vals}(\sigma_\ell) \), \( \sigma \) is the ets of an execution in \( P \). (Proof in the extended version [Pick et al. 2023b].)

**Symbolic Exploration Terminates.** Alg. 1 (without line 4) terminates for all (liftings of) P systems with finite executions. Note that if P systems have only finite executions then for every execution, there is a configuration where none of the event handlers for the enabled messages send new messages, resulting in a strictly decreasing number of messages in the buffer until the buffer becomes empty. From Theorem 5.5, every configuration explored and every message explored by Alg. 1 on the lifted P program have corresponding configurations and messages in an execution of the P program. In particular, Alg. 1 does not add any spurious messages to the global buffer. During exploration by Alg. 1, the global buffer will thus also have a decreasing number of messages until it becomes empty, at which point the condition on line 3 will be false, leading to termination.

**Example 5.6.** Consider running Alg. 1 on a configuration value summary \( c \) with global buffer \( B_c \) containing two messages with summaries \( m_1, m_2 \), both enabled under the guard \( g = D(m_1) = D(m_2) \). Assume that neither \( m_1 \) nor \( m_2 \) has a handler that sends a new message. Alg. 1 will terminate for this configuration value summary after running SCHEDULE STEP twice.

- **First SCHEDULE STEP.** Message value summaries \( m_1, m_2 \) will be in choices.
  - The **choose** rule introduces a fresh Boolean variable \( b_0 \) to distinguish between the choices, associating \( b_0 \) with \( m_1 \) and \( \neg b_0 \) with \( m_2 \).
  - \((m_1[b_0], (m_2[\neg b_0])\) are removed from the buffer \((c10)\), leaving \( m_1 \) in the buffer under guard \( g \wedge \neg b_0 \) and \( m_2 \) in the buffer under guard \( g \wedge b_0 \).
  - After restricting the resulting buffer with \((g \wedge b_0) \lor (g \wedge \neg b_0)\)\((c14)\), it does not change.

- **Second SCHEDULE STEP.** At this point, we still have two enabled messages in the buffer (at the end of the above step). Let \( m_1' = (m_1[g \wedge \neg b_0]) \), and \( m_2' = (m_2[g \wedge b_0]) \).
  - The **choose** rule introduces another fresh Boolean variable \( b_1 \) to distinguish between these choices, associating \( b_1 \) with \( m_1' \) and \( \neg b_1 \) with \( m_2' \).
  - \((m_1'[b_1], (m_2'[\neg b_1])\) are removed from the buffer \((c10)\), leaving \( m_1' \) in the buffer under guard \( g \wedge \neg b_0 \wedge \neg b_1 \) and \( m_2' \) in the buffer under guard \( g \wedge b_0 \wedge b_1 \).
  - After restricting the resulting buffer with \((g \wedge \neg b_0 \wedge b_1) \lor (g \wedge b_0 \wedge \neg b_1)\)\((c14)\), the guards in the messages in the buffer both become false. Hence, the buffer is empty, and Alg. 1 terminates.

### 5.4 Symbolic exploration with fixed-point detection

So far, we have considered systems with only finite executions. If we want to handle systems with infinite executions, we can still terminate and remain sound if they have fixed points in their
Algorithm 2 Fixed point detection using value summaries.

1: procedure CheckFixed(prevFrontiers, vs_c)
2:     coveredGuard = ⊥
3:     for vs ∈ prevFrontiers do
4:         if coveredGuard ⇒ D(vs_c) then coveredGuard ← coveredGuard ∨ GuardEqualUnder(vs, vs_c)
5:     else return coveredGuard
6:     return coveredGuard

executions, after which point no new behaviors of the system are explored. We can detect fixed
points by finding a case where a configuration in the execution repeats; exploring past this repeated
configuration will only yield more already-seen configurations.

When considering a configuration as repeating, we do not consider exact values of message
ids, as the values of messages’ unique id do not affect execution. As long as there is a bijection
between the message ids of one configuration and another such that applying this mapping to one
configuration results in the other, we consider them the same. Let sameC be a function over pairs of
configurations (c1, c2) that returns true if there exists a bijection map for c1 and c2, where replacing
each message id in c1 with the result of calling map on it results in a configuration equal to c2.

If we consider an explorer that explores a single execution at a time, to find fixed points, we
need to detect when there are c0, c1, ck in the execution so far c0 ⇒ c1 ⇒ ... ⇒ cn, where
sameC(c0, ck). This is similar to identifying covered nodes in program unwindings [Henzinger et al.
2002; McMillan 2006]: during exploration, for any node representing a state seen before on the
same path, the node is regarded as covered, and the subtree at the node is no longer explored.

For detecting fixed points of lifted P programs, we can lift this approach to operate over config-
uration value summaries instead and thus apply it to frontiers of configurations. Recall that SCI
Guards symbolically represent sets of sequences of nondeterministic choices that correspond to
paths in the execution tree of the program. Using the the lifted version sameCℓ of sameC, which
operates over configuration value summaries and returns a Boolean value summary, we can thus
detect the guards under which a fixed point has been found. Specifically, for a value summary vs
representing an earlier frontier and vs_c representing a later one, we use GuardEqualUnder(vs, vs_c)
to compute the guard under which configurations in vs_c are repeats of configurations in vs. We
define GuardEqualUnder as λx, y. getGuardFor(sameCℓ(x, y), true), where getGuardFor computes
the guard under which the first argument has the value of the second argument (as described in
§4.3). If GuardEqualUnder(vs, vs_c) does not return D(vs), then there are paths under which some
configuration in vs_c is not covered by (i.e., not included in) the set of configurations in vs.

Fixed-point detection in the lifted setting thus should check whether the current frontier value
summary vs_c contains only configurations that have occurred in previously-seen frontiers (stored in
the set of value summaries prev, Alg. 1) - if so, then the whole exploration has reached a fixed point.
This can be done as shown in Alg. 2, which computes the SCI Guard checkFixed by performing a
disjunction over the set of previous configurations.

Once all configurations in vs_c have been covered (line 5 in Alg. 2), a fixed point has been reached.
If they have not all been covered (checkFixed(prev, vs_c)  D(vs_c)), then exploration should and does
continue (see Alg. 1). This fixed point detection is enabled by the canonicity of value summaries
(requirement R1) and by including all nondeterministic (including scheduling) choices in the guards
(requirement R3). These features allow Alg. 2 to find fixed points that correspond to those in the
original program executions, allowing Alg. 1 to terminate for P programs that contain only finite
executions or those that have fixed points. This is stated in the following theorem, which is a
consequence of Theorem 5.5 and Alg. 2.

THEOREM 5.7. For any program P with lifted program P_l, if every full execution c_0 \xrightarrow{m_1} c_1 \xrightarrow{m_2} \ldots of P is finite or such that there exists i, k with i \neq k, where sameC(c_i, c_k), then when Alg. 1 with line 4 is run on P_l, it terminates. (Proof in the extended version [Pick et al. 2023b].)

In practice, the cost of fixed-point detection increases as greater depths are explored, since more BDD variables are introduced and prev grows larger. As a result, we use the fixed-point check in practice mostly when the set of possible configurations is small so that convergence is likely. Abstractions (described in §6.1) can help ensure faster convergence.

6 SYMBOLIC EXPLORATION WITH ABSTRATIONS AND REDUCTIONS

6.1 Abstract Value Summaries

Our approach can employ abstractions [Cousot and Cousot 1977; Cousot et al. 2013; Fähndrich and Logozzo 2010; Flanagan and Qadeer 2002; Graf and Saitd 1997] to convert large or infinite number of concrete configurations into a smaller and finite number of abstract configurations. As noted previously [Sen et al. 2015], one merit of a value summary representation is the ability to replace the value component of a primitive value summary with any representation – concrete values that occur when executing the program or abstractions of sets of such values.

Let C be a set of concrete values and A be an abstract set for these values, such that there exists a map \alpha : P(C) \rightarrow A that maps each element of P(C) to the element of A that abstracts it. We refer to value summaries whose primitive value summaries’ value components are all in C as concrete value summaries and those whose value components are all in A as abstract value summaries.

Abstract Values. It is straightforward to abstract a concrete value summary to an abstract value summary by traversing the recursive structure of a given value summary and, for each value component c, applying the map \alpha to the singleton set \{c\}.

Abstract Semantics. The semantics of any program defined over concrete value summaries can be lifted to work over the abstract domains in a manner similar to other settings. The semantics are as in §5.1, except that each concrete value (resp. value summary) is replaced by an abstract value (resp. abstract value summary). While we do not consider this in-depth here, for increased precision, we can perform special handling for branches on nondeterministic values whose nondeterminism is captured in the abstraction rather than in SCI Guards. The extended version has a case study on using abstractions to verify infinite-state distributed systems [Pick et al. 2023b].

6.2 Partial-Order and Other Reductions as Filters

To extend our symbolic explorer with capabilities to perform reduction, we leverage Filters functions (C \rightarrow P(M) \rightarrow P(M)). Each filter, when applied to a configuration and a set of enabled messages, returns a subset of the enabled messages that can be scheduled at that step.

Using Filters. Let ApplyFilters({f_0, \ldots, f_n}, c) = (f_0 c) \circ \cdots \circ (f_n c)({y \mid enabled(m, c)}), where \circ denotes function composition. This function can be used to apply a set of Filters to the set of messages in a given configuration. Filters can be incorporated into the lifted P semantics by amending the SCHEDULE STEP rule. Filters should be added to the context of the judgment and the premise m \in \ell(\lambda x. ApplyFilters(Filters, x))(c) should be added to the rule.

Reductions and pruning. We can implement several reductions using filters. Left movers from Lipton’s theory of reductions [Lipton 1975] can be implemented with a function LM that guarantees that left movers are scheduled before any other enabled messages: LM(c)(ms) returns \{left\} for some left \in ms if left is a left mover and otherwise returns ms. Persistent-set-based POR can be implemented with a filter function that removes messages outside of the persistent set. In §7, we make use of a persistent-based POR called PRed that we developed for P; the filter function is given by \lambda c. \lambda ms.\{m \in ms \mid \forall m' \in B_c, m' \not\in c, m\}. This function removes from the enabled set all messages
for which another message in the global message buffer is lower according to the < relation, an extension of send-order (SO) with the happens-before relation. See the extended version for more information about PRED [Pick et al. 2023b]. Monotonic POR [Kahl et al. 2009] can be implemented similarly to persistent-set-based POR, and a variant of sleep-set-based POR [Godefroid 1990] can be implemented by using information across different executions being explored. We can also use filtering functions to prune based on external analyses, e.g., of symmetries in the system.

7 EVALUATION

Implementation. We implemented our approach in the PSYM tool as an extension of the open-source P framework. Our implementation consists of three parts: (1) a Java library that implements a symbolic runtime with support for creating and manipulating value summaries (as described in §4.3); (2) a P compiler extension to generate Java code that symbolically represents lifted P program using the value summary implementations in the symbolic runtime (thus lifting the P program over value summaries as described in §5.1); and finally, (3) a systematic explorer that implements Alg. 1. Guards in value summaries are represented using the PJBDD library [Beyer et al. 2021]. Unless otherwise stated, experiments were run on a Linux machine with Intel(R) Xeon(R) CPU @ 2.30GHz, 16 cores, and 200GB main memory.

Research Questions. Our evaluation seeks to answer the following research questions:

Q1. How does PSYM compare against a state-of-the-art model checker for verifying distributed protocols?

Q2. Does PSYM succeed in verifying P models from previous papers?

Q3. Does PSYM scale to verify distributed protocols from real-world industrial systems created by developers (not by the authors)?

Q4. Does PSYM succeed in verifying infinite state systems and how does it compare against other approaches for systematic exploration?

We use the PRED partial-order reduction in PSYM in all experiments except in Q1.

Characteristics of models. Other than TLA+ models (in §7.1 for Q1), each P model we consider has all types of nondeterminism: asynchronous message interleaving, dynamic machine creations, and data or input nondeterminism (e.g., node failures). All the models are closed and have a finite number of configurations, i.e., for each system, there are a finite number of processes, and the system takes finite inputs. Also, for each model (other than those in §7.4 for Q4, which have infinite executions), every execution is terminating and the total number of executions is finite.

7.1 Q1: Comparison with TLC (model checker for TLA+)

P no longer supports the Zing [Andrews et al. 2004] model checking backend used in previous approaches [Desai et al. 2013b]; instead it relies on Coyote [Microsoft Coyote 2022] for randomized testing of P models and has no support for verification. To demonstrate the efficacy of our approach, we would like to compare against a state-of-the-art model checker for distributed systems (not built by the authors). TLA+ [Lamport 2002] is a popular modeling language and is a standard in industry [Newcombe 2014] and academia [TLA+ 2023] for model-checking distributed protocols. TLC [Yu et al. 1999] is an explicit state model checker for TLA+ models and has evolved over the years with optimizations to scale model checking to complex models.

To evaluate the efficacy of our approach, we compare PSYM with TLC on several open-source TLA+ models, which we manually translated into P programs. Note that P and TLA+ support different models of computation; our objective with this experiment is to compare the search exploration techniques implemented by PSYM and TLC. We confirmed through manual review that the P models have same nondeterminism as allowed in the TLA+ models. These models include the following: Hybrid Logical Clocks [Demirbas 2017a; Kulkarni et al. 2014] with a parameterizable maximum time (\(\text{maxT}\)); the decentralized consensus algorithm proposed by Ben-Or [Ben-Or 1983; Demirbas...
Table 1. Comparison between TLC and PSYM, with 12GB memory (max) and 15-minute timeout.

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>TLC</th>
<th>PSYM</th>
<th>Benchmark</th>
<th>TLC</th>
<th>PSYM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chang-Roberts (N=5)</td>
<td>7s</td>
<td>1s</td>
<td>2PCwithBTM</td>
<td>17s</td>
<td>10s</td>
</tr>
<tr>
<td>HLC (MaxT=3)</td>
<td>18s</td>
<td>3s</td>
<td>BenOr (MaxRound = 2)</td>
<td>24s</td>
<td>7s</td>
</tr>
<tr>
<td>HLC (MaxT=4)</td>
<td>24s</td>
<td>8s</td>
<td>BenOr (MaxRound = 3)</td>
<td>1m20s</td>
<td>1m11s</td>
</tr>
<tr>
<td>HLC (MaxT=5)</td>
<td>42s</td>
<td>24s</td>
<td>Paxos (simple, STOP=3, M=2, MAXB=3)</td>
<td>3m20s</td>
<td>3m21s</td>
</tr>
<tr>
<td>Streamlet Blockchain</td>
<td>Timeout</td>
<td>5s</td>
<td>Paxos (full, STOP=1, M=9, MAXB=10)</td>
<td>Timeout</td>
<td>6m54s</td>
</tr>
</tbody>
</table>

2019]; a version of two-phase commit with a transaction manager [Demirbas 2017b,c]; a simplified version and the full version of the Flexible Paxos distributed consensus protocol [Howard et al. 2016], for which the open-source TLA+ model has four proposers [fpa 2022] and a parameterizable number of values to reach consensus on (STOP), number of proposers (M), and maximum number of ballots (MAXB); the Streamlet blockchain protocol [Chan and Shi 2020]; the Chang-Roberts ring leader election algorithm [cha 2021; Chang and Roberts 1979] with a parameterizable number of nodes (N). We compare PSYM against TLC when run with 1 worker thread (PSYM is single-threaded), and limit both tools to 12GB memory. The results are shown in Table 1; experiments were run on a Macbook Pro with an M1 processor, 16GB RAM with a 15-minute timeout.

For a fair comparison to TLC (which does not implement POR), we do not use the PRED POR in PSYM for these experiments – thus, our results are due to use of symbolic representations and targeting additional redundancies due to overlapping transitions. On the other hand, TLC uses state hashing to avoid re-exploring from previously-seen configurations. In contrast, we do not currently use any state hashing in PSYM (and plan to implement it in future work). In PSYM, merging allows us to avoid re-exploration from the same configurations in the same frontier.

For most benchmarks, there are many distinct transitions that are overlapping, leading PSYM to outperform TLC. In both Paxos and Streamlet, there are many instances of the same process that perform the same or similar computations (e.g., finding the maximal notarized chain in Streamlet). PSYM was able to take advantage of these overlapping transitions to complete verification, whereas TLC timed out. For the remaining benchmarks, PSYM showed a runtime improvement of 2.5X on average (geometric mean) compared to TLC. However, some benchmarks have configurations that are the same at different depths in the execution tree, where PSYM may perform redundant work.

7.2 Q2: Evaluation on P programs from previous papers

We evaluated PSYM on common distributed protocols available in the P GitHub repository, used as benchmarks in previous publications [Deligiannis et al. 2015; Desai et al. 2015; Liu et al. 2019]. These are: (consensus) Two-Phase Commit [Gray and Lamport 2006] and Paxos [Lamport 2001]; (leader election) Token Ring [Lynch 1996], Failure Detector, and Bounded broadcast [Liu et al. 2019]; and German cache coherence [Pnueli et al. 2001]. Our Two-Phase Commit benchmark has two clients and two participants, where each client tries to perform two read and two write transactions; Paxos runs 4 rounds and has two proposers, which each propose a single value, and three acceptors. The Token Ring is of size 4. We model failures for these benchmark. While some of these benchmarks (e.g., German, Token Ring) exhibit symmetry, we did not use symmetry-based pruning filters.

Table 2 shows the results of running PSYM on these benchmarks, where we report the number of lines of P code (LoC), the number of concurrently executing state machines (#M), the time taken (Time), and the max memory consumed (Mem). The #Steps column reports the number of (lifted) event handler invocations made during exploration, where the number in parentheses reports the number of event handler invocations that would have occurred without handling messages with the same event handlers in one step. This demonstrates the benefits from identifying overlapping transitions. For example, in Two-Phase Commit, many different participant machines send messages to a single coordinator machine. Because all participants send only two kinds of messages to the
Table 2. Benchmarking PSYM on standard distributed protocols

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>LOC (P)</th>
<th>#M</th>
<th>Time</th>
<th>#Steps w/ (and w/o) overlapping transitions</th>
<th>Mem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Token Ring</td>
<td>164</td>
<td>5</td>
<td>2s</td>
<td>143 (243)</td>
<td>27 MB</td>
</tr>
<tr>
<td>BoundedAsync</td>
<td>96</td>
<td>4</td>
<td>11s</td>
<td>534 (534)</td>
<td>45 MB</td>
</tr>
<tr>
<td>German</td>
<td>283</td>
<td>5</td>
<td>44s</td>
<td>244 (300)</td>
<td>860 MB</td>
</tr>
<tr>
<td>Failure Detector</td>
<td>189</td>
<td>7</td>
<td>38s</td>
<td>277 (435)</td>
<td>1.4 GB</td>
</tr>
<tr>
<td>Two-Phase Commit</td>
<td>284</td>
<td>7</td>
<td>57m22s</td>
<td>27 (2643)</td>
<td>13 GB</td>
</tr>
<tr>
<td>Paxos</td>
<td>241</td>
<td>8</td>
<td>2h7s</td>
<td>931 (2143)</td>
<td>18 GB</td>
</tr>
</tbody>
</table>

Table 3. Verification of industrial case studies using PSYM (L: Lipton reduction, S: Symmetry based pruning)

<table>
<thead>
<tr>
<th>Case Study</th>
<th>LOC (P)</th>
<th>#M</th>
<th>PSYM</th>
<th>PSYM with custom filters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Time</td>
<td># Step (Mem.)</td>
</tr>
<tr>
<td>OTA</td>
<td>1103</td>
<td>14</td>
<td>678 (33 GB)</td>
<td>15m6s</td>
</tr>
<tr>
<td>Data migration</td>
<td>959</td>
<td>22</td>
<td>5332 (48 GB)</td>
<td>5h23s</td>
</tr>
<tr>
<td>Data replication</td>
<td>1439</td>
<td>16</td>
<td>5132 (59 GB)</td>
<td>3h1m14s</td>
</tr>
<tr>
<td>Conflict resolution</td>
<td>1202</td>
<td>13</td>
<td>2182 (122 GB)</td>
<td>5h1m</td>
</tr>
</tbody>
</table>

coordinator, only (at most) two lifted event handler executions are needed to handle the messages sent to the coordinator. We found that in most of these distributed protocols, the configurations reached after exploring different interleavings of messages, still have many equivalent components, leading to efficient handling of redundancies in overlapping transitions.

7.3 Q3: Verifying industrial case studies: distributed storage and database protocols

We used PSYM to verify some real-world industrial distributed protocols from the storage, IoT, and database systems at Amazon Web Services (AWS); these protocol models were implemented by expert engineers in AWS, and have the characteristics described earlier with failures and asynchrony. We only applied PSYM and helped developers identify opportunities to apply sound reductions. We used PSYM to verify the correctness of four industrial case studies: (1) a distributed protocol used at Amazon S3 for reliable data migration; (2) a distributed protocol used at Amazon S3 for durable data backup; (3) The AWS Over the Air (OTA) protocol used to update an IoT device firmware reliably and securely; and (4) a multi-region distributed transaction conflict resolution protocol used for a database service at AWS. We asserted safety properties such as consistency for the distributed transaction conflict resolution protocol and reliable file transfer for OTA. PSYM verified finite instances of these models, where other techniques we tried failed (see discussion in §8). Finite instances came from bounding the number of processes and inputs but not depth. For example, PSYM verified the conflict resolution protocol for 2 geographical regions, each with 3 replicas, and 2 clients concurrently sending 3 non-deterministic commands (insert, delete, update) to any region.

Table 3 reports the results of running PSYM on these benchmarks both with and without additional reductions. PSYM was able to successfully verify the correctness of these benchmarks, demonstrating that it can be used as a verifier for realistic systems. These results also demonstrate the effectiveness of incorporating custom filters into PSYM. Specifically, the OTA protocol makes several synchronous interactions with other components, and the response messages for these interactions are left movers based on Lipton’s theory of reductions [Lipton 1975]. The data migration and replication protocols use a centralized locking service to gain consensus, and the release message from this service is also a left mover. We also found that certain operations (messages) can be safely pruned because of symmetry. Thus, adding Lipton’s left mover (L) and symmetry-based pruning (S) filters helped reduce the number of interleavings, which led to smaller sizes of the representations and hence faster verification times. The ability of PSYM to integrate these filters proved effective.
7.4 Q4: Verifying infinite-state systems and comparison with other approaches

We successfully applied PSYM with the fixed-point detection enabled on two variants of distributed key-value store implemented on top of Paxos and Two-Phase commit protocols. These benchmarks had infinite-state systems with infinite-length executions. We use abstractions to convert them into finite-state systems but still with infinite-length executions, which are handled by fixed-point detection. Evaluation details, as well as a discussion comparing PSYM with other techniques like Dynamic POR [Abdulla et al. 2014; Aronis et al. 2018; Flanagan and Godefroid 2005], are in the extended version [Pick et al. 2023b].

8 RELATED WORK

Verification and Invariant Discovery for Distributed Systems. Many approaches use deductive proof techniques to formally verify distributed systems [Hawblitzel et al. 2015; Padon et al. 2016; Wilcox et al. 2015]. They can prove correctness of distributed systems for unbounded inputs but require users to provide complex inductive invariants.

Model checking provides an algorithmic approach for verifying either a model (e.g., SPIN [Holzmann 1997], Zing [Andrews et al. 2004], TLC [Yu et al. 1999]) or an implementation (e.g., Verisoft [Godefroid 1997], JPF [Visser and Mehltz 2005], CHESS [Musuvathi and Qadeer 2007]), but suffers from state space explosion. Several sequentialization techniques [La Torre et al. 2009; Lal and Reps 2008] reduce verification of concurrent or distributed programs to verification of sequential programs, but the resulting sequentialized programs are highly nondeterministic. State caching (used e.g., in TLC [Yu et al. 1999]) addresses some scalability issues by avoiding re-exploration from redundant configurations. We provide an approach that additionally avoids repeating redundant computations in updates due to overlapping transitions.

Before developing our proposed approach, we experimented with using bounded model checking [Biere et al. 1999]. We hand-coded the Two-Phase Commit protocol in the open-source tools UCLID5 [Seshia and Subramanyan 2018] and JKind [Gacek et al. 2018]. Both tools failed to verify the protocol within 2 hours. While PSYM has a large memory footprint typical for BDD-based tools, we are encouraged that it completes verification successfully on many challenging benchmarks.

Recent efforts in automated invariant discovery for distributed protocols generalize from small instances of protocols to learn invariants for all instance sizes [Ma et al. 2019; Yao et al. 2021]. Our explorer could be used as a subprocedure in such techniques e.g., as a model-checker for a technique such as in J4 [Ma et al. 2019], or as a systematic explorer for a data-driven technique such as in DistAI [Yao et al. 2021]. It may also be used to generate additional invariants as candidates for generalization in either kind of technique.

Symbolic and Abstraction Techniques. Symbolic model checking [Burch et al. 1992] has been successfully applied to many problems but does not scale well when applied to distributed systems due to state space explosion. Recent efforts include a symbolic model checker for TLA+ [Konnov et al. 2019], which improves scalability of invariant checking but does not show clear advantages over TLC for model checking safety properties; it is much slower than TLC on distributed benchmarks such as two-phase commit and Paxos. (We thus compared only against TLC in our experiments.)

Symbolic and concolic execution have been somewhat effective in addressing state space explosion due to input and control nondeterminism by focusing on one control path at a time [Cadar et al. 2008; Farzan et al. 2013; Godefroid et al. 2005; Sen and Agha 2006b]. However, this leads to path explosion for reasonable coverage. Several techniques have been proposed to handle path explosion by state merging [Anand et al. 2008; Kuznetsov et al. 2012; Sen et al. 2015; Tørlik and Bodik 2014]. We follow in the steps of MultiSE [Sen et al. 2015], extending their guards to handle scheduling choices as well as adapting their symbolic representation. To the best of our knowledge, no existing approaches have proposed handling scheduling choices symbolically or merge state in distributed
systems as we do. One work [Sen and Agha 2006a] combines concolic execution with POR and uses backtracking to explore both different scheduling choices and control-flow paths but does not involve a symbolic encoding of scheduling choices. A more recent effort [Schemmel et al. 2020] also combines POR and symbolic execution but also does not represent scheduling nondeterminism symbolically. Our fixed point check is an adaptation of an existing covering approach for model checking [McMillan 2006] to work on configuration value summaries.

Abstraction has been widely applied in many domains to help combat state-space explosion and finitize infinite-state systems [Cousot and Cousot 1977; Cousot et al. 2013; Fähndrich and Logozzo 2010; Flanagan and Qadeer 2002; Graf and Saidi 1997]. We leverage prior work on abstraction to achieve similar benefits, allowing our technique to scale even further.

**POR and Symmetry Reductions.** (Dynamic) POR [Abdulla et al. 2014; Aronis et al. 2018; Flanagan and Godefroid 2005; Kahl et al. 2009; Nguyen et al. 2018; Peled 2018] and symmetry reduction [Iosif 2002] address state space explosion problem by taking a control-centric view during the exploration of a distributed system's state space. They do not consider the state of the system during the exploration.

Our approach provides a complementary data-centric view that allows our explorer to recognize not only redundant interleavings (as in POR) and configurations (as in state caching [Holzmann 1997]) but also redundancies in overlapping transitions. As seen previously, we can integrate POR and symmetry reductions with our proposed techniques, but there are limitations in combining our approach with Dynamic POR techniques [Abdulla et al. 2014; Aronis et al. 2018; Flanagan and Godefroid 2005] that rely on a depth-first order of exploration. See the extended version for a comparison with a tool that uses optimal dynamic POR [Pick et al. 2023b].

**Systematic Testing of Distributed Systems.** Systematic testing has successfully uncovered deep, hard-to-find bugs in distributed systems. Systematic testing techniques often use prioritized search [Deligiannis et al. 2015; Desai et al. 2015; Leesatapornwongsu et al. 2014; Mukherjee et al. 2020] and stratified random testing [Deligiannis et al. 2015; Jepsen 2021; Killian et al. 2007; Majumdar and Niksic 2017; Ozkan et al. 2018] to guide exploration and typically do not aim to exhaustively explore behaviors as we do here. For P programs in particular, researchers have proposed bounded exploration based on delay-bounding [Desai et al. 2015] and reductions based on almost-synchronous invariants [Desai et al. 2014]; our work is the first to use POR specific to P semantics (in the form of PRED). Recent work [Liu et al. 2019] presents sound partial abstract transformers for verification of P programs. This approach, like ours, computes fixed points over abstract states, but it does not use a fine-grained configuration representation nor identify overlapping transitions. Furthermore, it is unclear how to combine POR with this approach.

9 CONCLUSIONS

We presented a novel approach for scalable stateful exploration of distributed systems implemented in P. Our approach leverages a novel canonical fine-grained symbolic representation of distributed system configurations, yielding an explorer that implicitly recognizes equivalent configurations and reduces redundancies due to overlapping transitions. Our explorer is designed so that scalability can be further increased via abstractions and various reductions. Evaluation shows that our tool Psym outperforms a state-of-the-art stateful explorer and can successfully verify P models of common distributed protocols and challenging industrial case studies. Future work includes the use of our approach in invariant discovery techniques and the implementation of state caching in Psym.

10 ACKNOWLEDGMENTS

We would like to thank Aman Goel, Cambridge Yang, William Brandon, and Eric Ge who contributed to Psym as well as our anonymous reviewers for their valuable suggestions. This work was supported in part by the National Science Foundation under Grant # 2127309 to the Computing Research Association for the CIFellows project, NSF-1837030, and an Amazon research award.
11 SOFTWARE AVAILABILITY

The version of Psym used to produce the results in the paper is available on Zenodo along with instructions on how to run some of the experiments in the paper [Pick et al. 2023a].

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